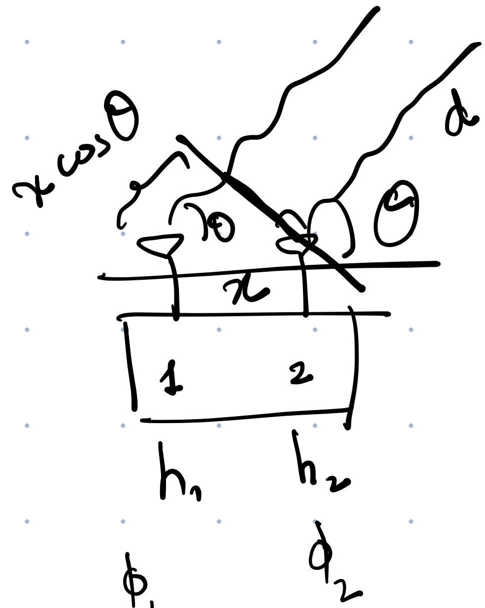
# CS 598, WSI : LECTURE 9

- Angle of arrival : Recap
- Multipath Profile
- Antenna array
- Time-of-flight
- Distance & Chronos.

# AoA: Recap




$$\phi_2 = -\frac{2\pi}{\lambda} d \mod 2\pi$$

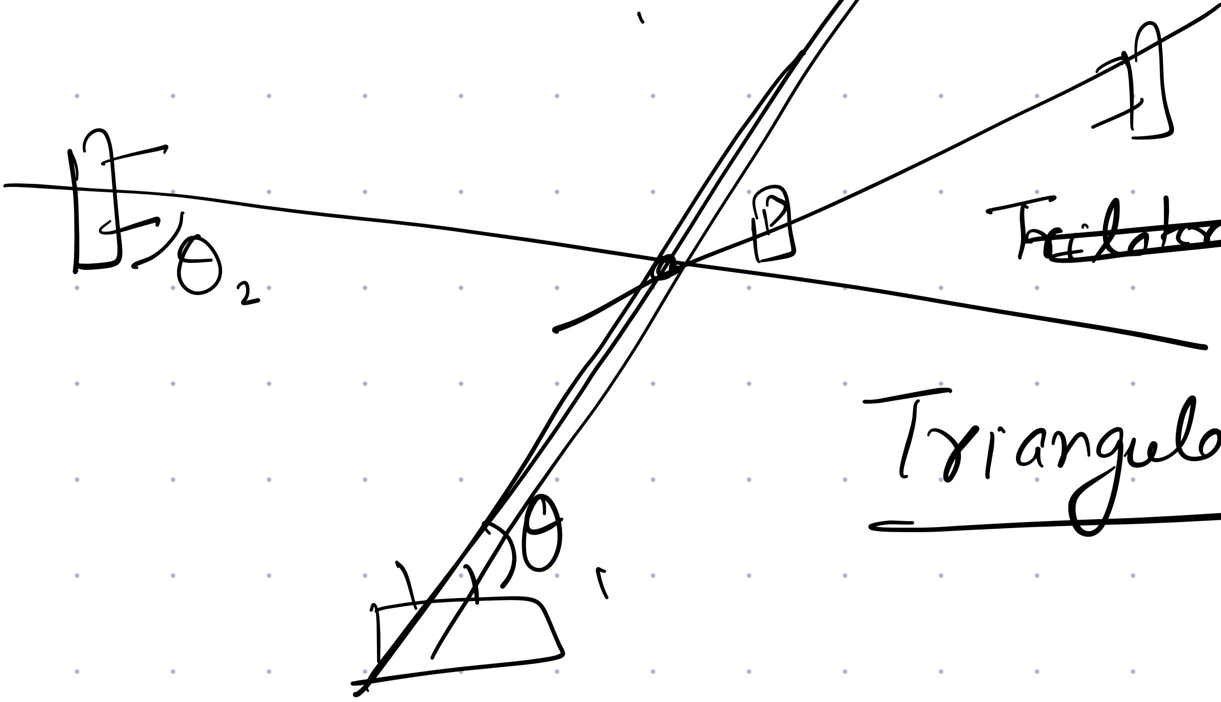
$$\phi_1 = -\frac{2\pi}{\lambda}(d + x\cos\theta) \mod 2\pi$$

$$\phi_2 - \phi_1 = \frac{2\pi}{\lambda} x \cos\theta \mod 2\pi$$

$\Downarrow$

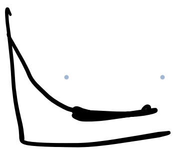
multiple solutions

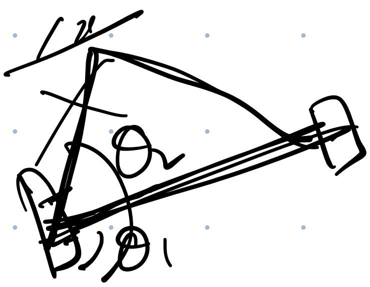
if $x \leq \lambda/2$, then unique solution for $\theta$.




~~Trilateration~~

## Triangulation

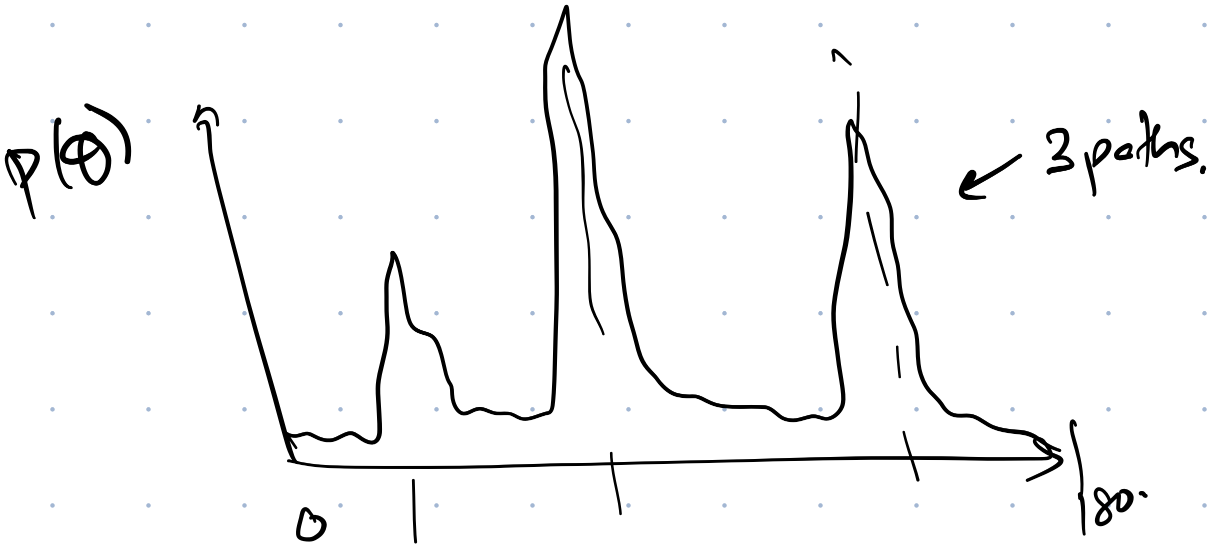
| Pros  | Cons |
| --- | --- |
| • Uniform error | • Multipath |
| | • Multiple access points |
| | • Phase |




Can I separate multipath?

Which of these paths is the direct path?

# MULTIPATH PROFILE



$$\Phi_2 = -\frac{2\pi}{\lambda} d \mod 2\pi$$

$$\phi_1 = -\frac{2\pi}{\lambda}(d + x\cos\theta) \mod 2\pi$$

$$h_1 = a\, e^{-j\frac{2\pi}{\lambda}(d + \underbrace{x \cos\theta}_{})}$$ ↳ 30°

$$h_2 = a\, e^{-j\frac{2\pi}{\lambda} d}$$

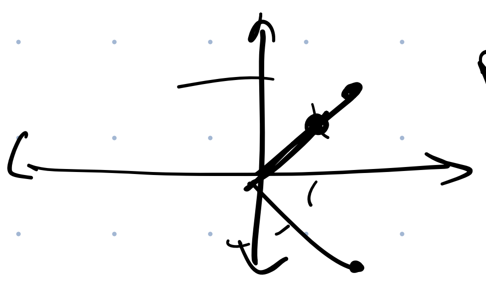
$$P(\theta) = h_1 + h_2 e^{-j \frac{2\pi}{\lambda} x \cos\theta}$$

$$P(\theta) = h_1 + h_2 e^{-j \frac{2\pi}{\lambda} x \cos\theta'}$$

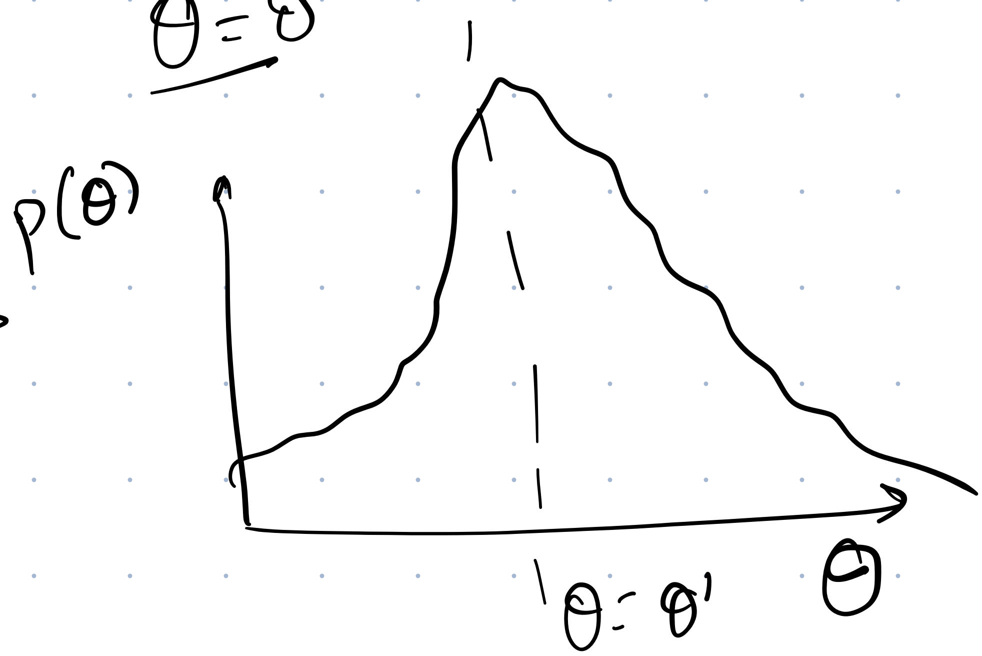
$\theta'$

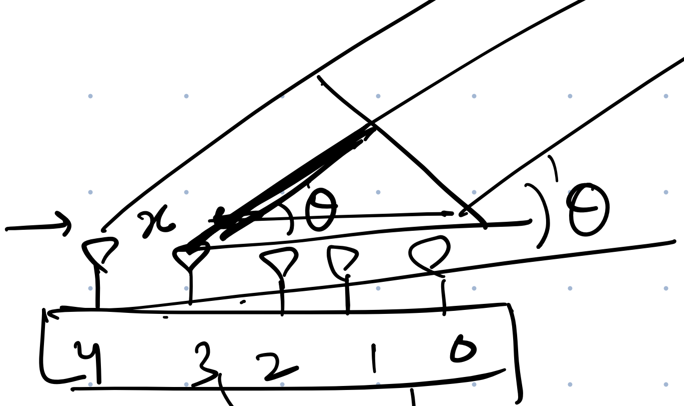
$$= a e^{-j \frac{2\pi}{\lambda}(d + x\cos\theta')} \times + a e^{-j \frac{2\pi}{\lambda}(d + x\cos\theta')} \times$$

$\theta \neq \theta'$



$\theta = \theta'$





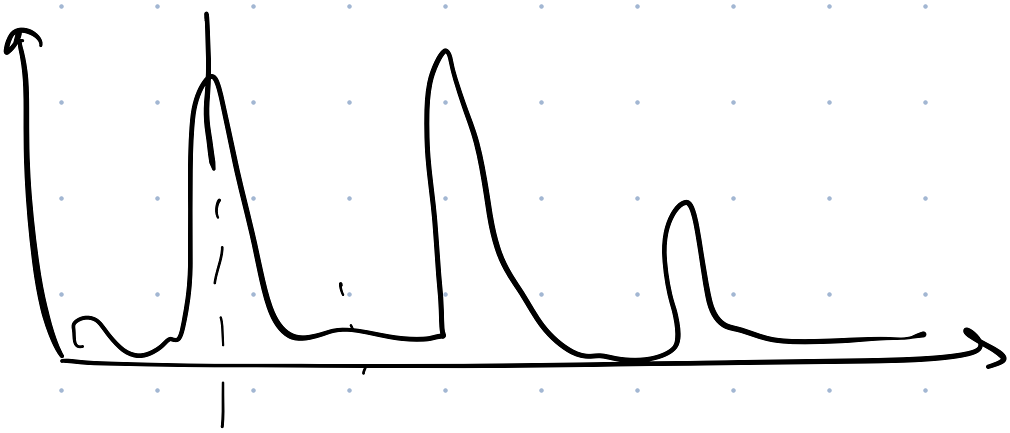

0 to 180°

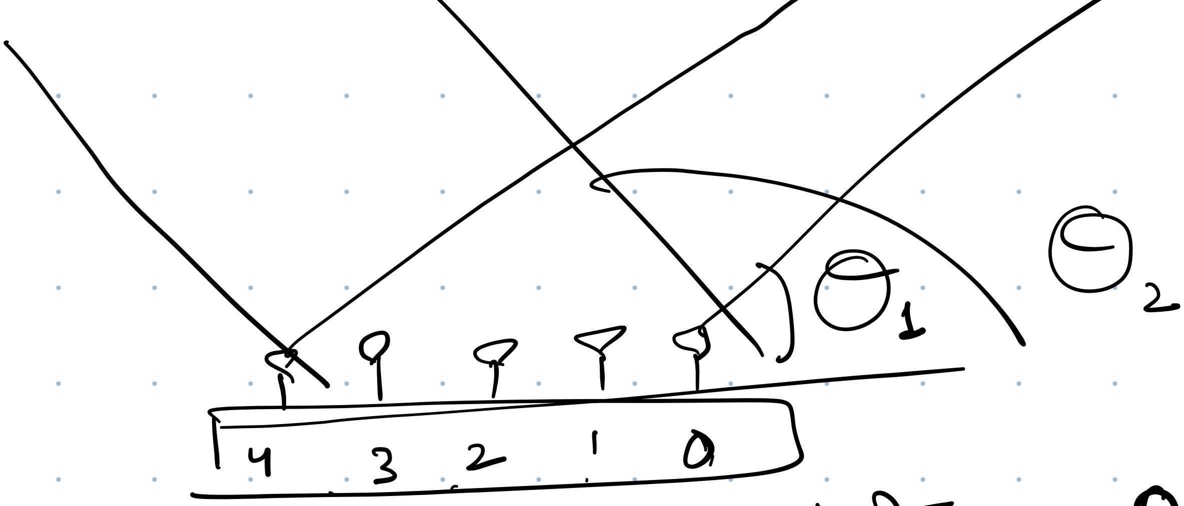
0 to π radi

$$\phi_4 - \phi_1 = -\frac{2\pi}{\lambda}(3x)\cos\theta'$$

$$\phi_i - \phi_0 = -\frac{2\pi}{\lambda}(ix)\cos\theta' \mod 2\pi$$

$$P(\theta) = \sum_i h_i e^{+j\frac{2\pi}{\lambda} i x \cos\theta}$$



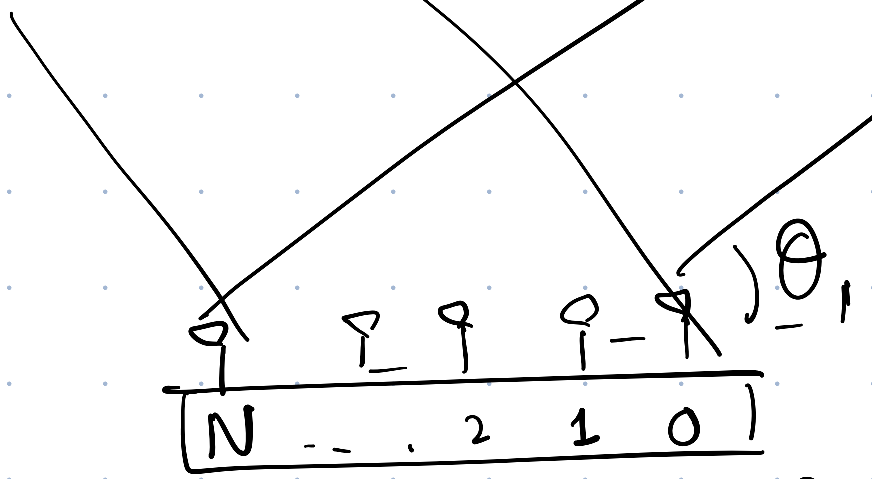



$$p(\theta) = \sum_i h_i \, e^{j\frac{2\pi}{\lambda} x \cos\theta}$$

$\theta = \theta_1 \Rightarrow \theta_1 \sim p(\theta_1)$, $\theta_2 \simeq 0$

$\theta = \theta_2 \Rightarrow 0$, $p(\theta_2)$

$\forall \theta \neq \theta_1$ & $\theta \neq \theta_2 \Rightarrow \simeq 0$


$$h_i = h_0 \, e^{-j\frac{2\pi}{\lambda} i x \cos\theta_1}$$

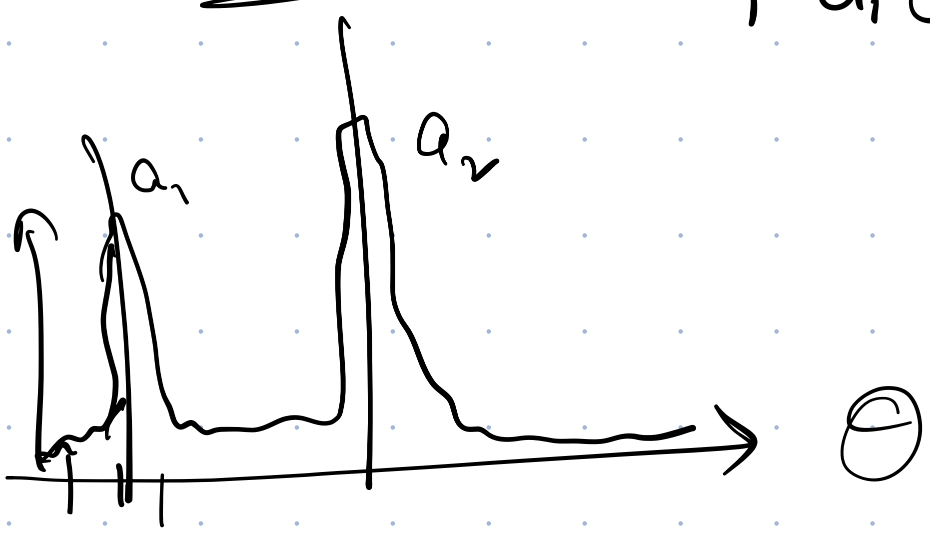
$$h_0 = a_0 e^{-j\frac{2\pi}{\lambda} d_0} + a_1 e^{-j\frac{2\pi}{\lambda} d_1}$$

$$h_i = a_0 e^{-j\frac{2\pi}{\lambda}(d_0 + i x \cos\theta_0)} + a_1 e^{-j\frac{2\pi}{\lambda}(d_1 + i x \cos\theta_1)}$$

$$\{h_0 + h_i e^{j\frac{2\pi}{\lambda} i x \cos\theta}\} = P(\theta)$$

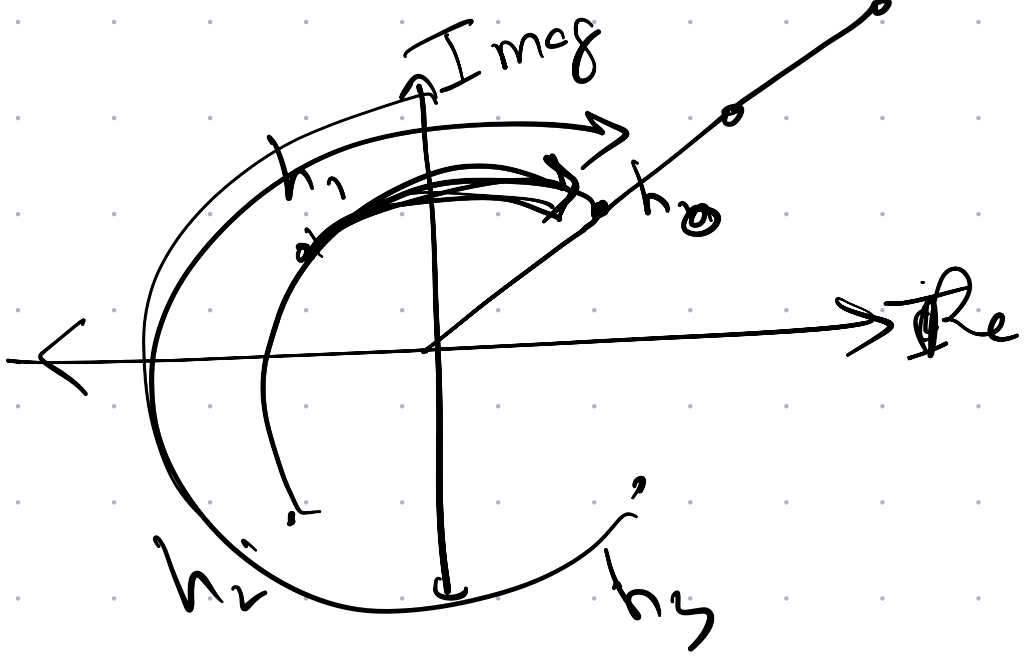
$\theta = \theta_0$

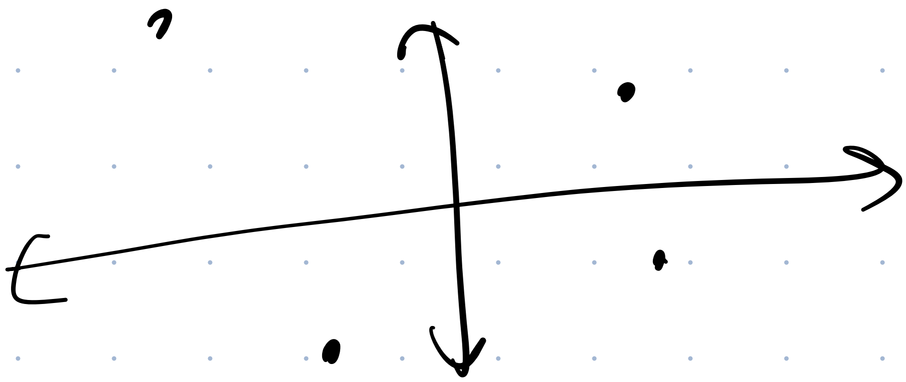
$$P(\theta) = 2a_0 e^{-j\phi} + a_1(e^{j\phi_1} + e^{j\phi_2}) + e^{j\phi_3} + e^{j\phi_3}$$



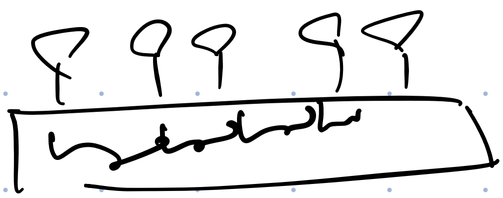
$$P(\theta) = \left| \sum_{i} h_i \, e^{j\frac{2\pi}{\lambda} i x \cos\theta} \right|$$

↳ a



# Antenna Array: Tradeoffs



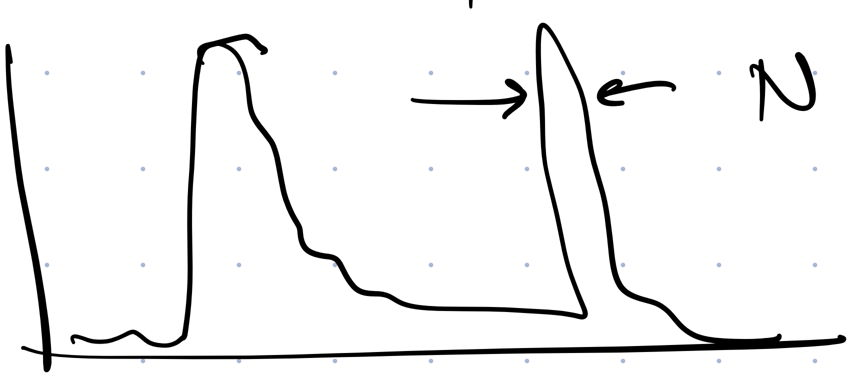
Uniform linear arrays.

$x = \lambda/2$

No. of antennas ;

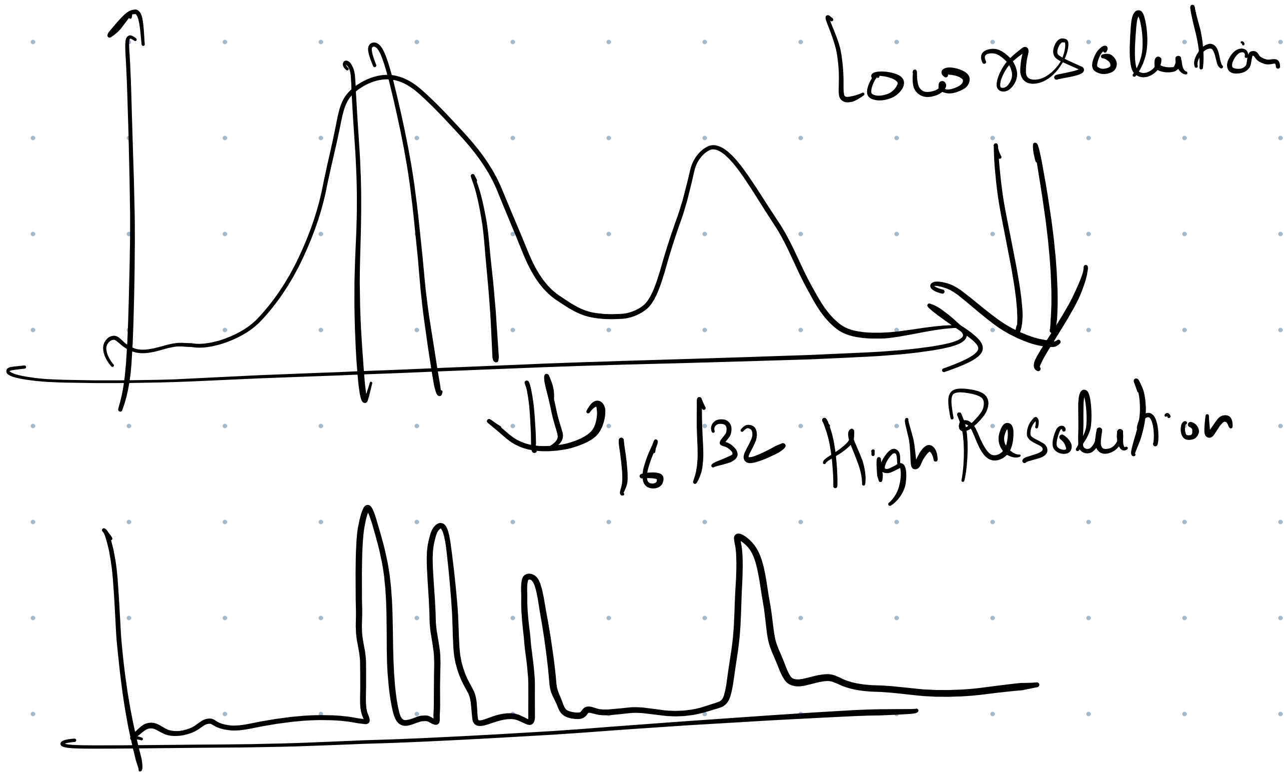
N antennas, separated by $\lambda/2$ each,

then each peak is approx. $\frac{\pi}{N}$ wide



N bigger $\Rightarrow$ peaks narrower.

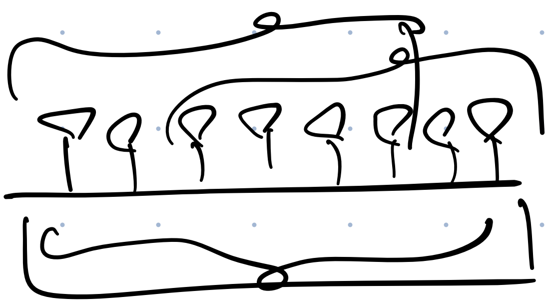
2 antenna


Low resolution
16/32
High Resolution

Q. How do you find the direct path?

→ Pick the strongest path as the direct path?

→



P(θ)

→ "shortest" ⇒ direct

↳ d

# Time-of-flight


$c = 3 \times 10^8$ m/s

$$d = ct$$

distance = speed of light × time of flight

$$t = (t_2 - t_1)$$

clocks are not synchronized,

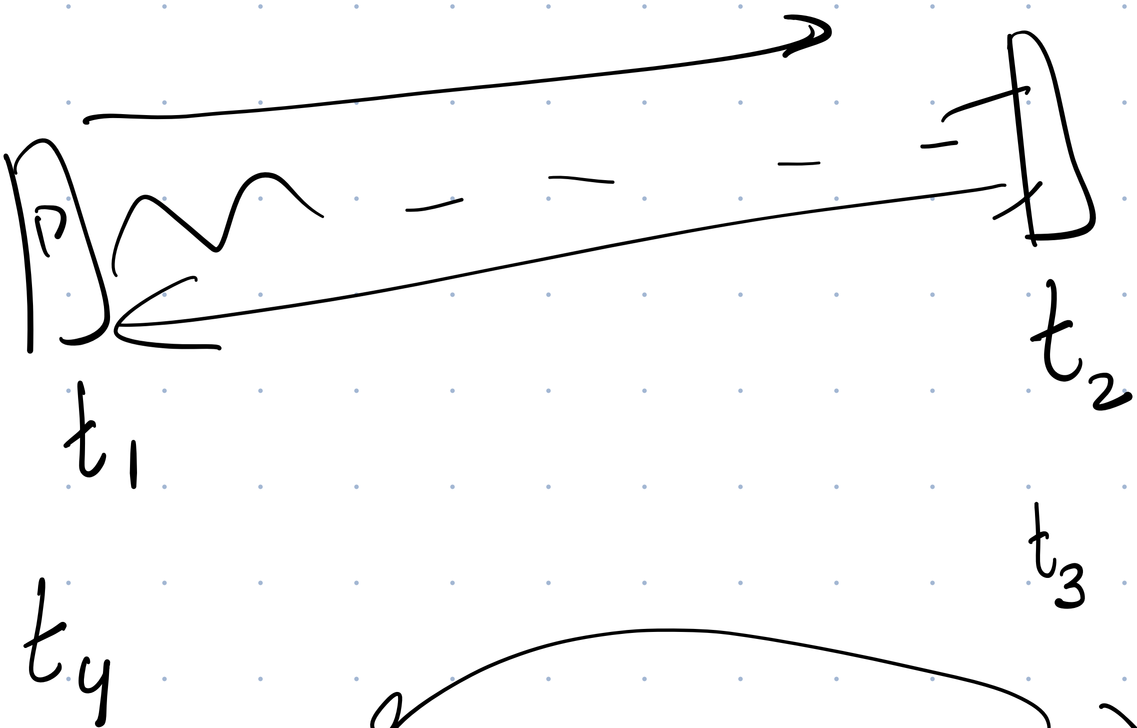
$c = 3 \times 10^8$ m/s

5 m away.

$$tof = \frac{5}{3\times10^8} m = \frac{16 ns}{} = 16\times10^{-9} s$$

1 μs error in clock ⇒ 300 m error in distance.

# FTM: Fine Time Measurements




$$t_{of} = \frac{(t_2 - t_1) + (t_4 - t_3)}{2}$$

$$= \frac{\overbrace{(t_2 - t_3)}^{\text{on the same clock}} + \overbrace{(t_4^{+\delta} - t_1^{+\delta})}^{\text{on the same clock}}}{2}$$

new Wi-Fi protocol

$t_1, t_2, t_3, t_4$ ← precise
ns level.

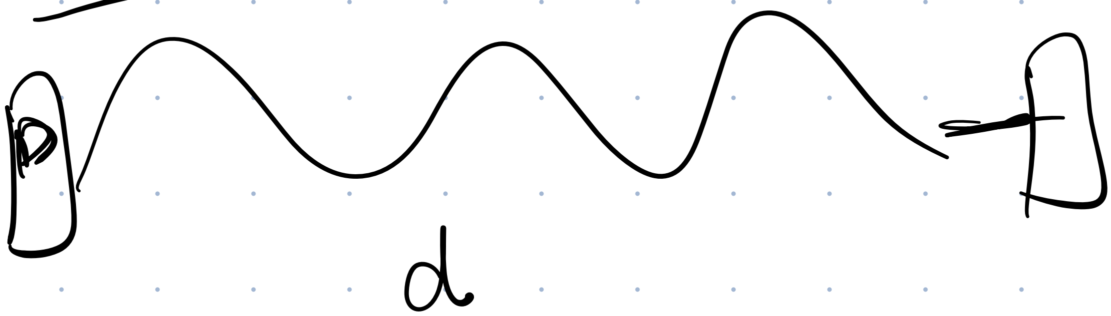
Multipath

$$40\,MHz \simeq \frac{1}{40 \times 10^6}\,s \simeq \frac{1000}{40}\,ns$$

$$\simeq 25\,ns$$

# Distance & Chronos

## Phase



$$\phi = -\frac{2\pi}{\lambda} d \mod 2\pi$$

$\lambda$ ↗ 5 cm.

$$\phi = -\frac{\pi}{5}$$

$$-\frac{\pi}{5} = -\frac{2\pi}{5} d \mod 2\pi$$

$$\left\{ d = \frac{1}{2}\text{cm} \mod 5\text{cm} \right\}$$

$\{0.5\text{ cm},\ 5.5\text{ cm},\ 10.5\text{ cm},\ 15.5\text{ cm}\}$

$$\phi = -\frac{2\pi}{\lambda} d \bmod 2\pi$$

$$-\frac{\lambda\phi}{2\pi} = d \bmod \frac{2\pi\lambda}{2\pi}$$

$$d = -\frac{\lambda\phi}{2\pi} \bmod \lambda$$

$$\phi = \frac{\pi}{5} \qquad \lambda = 5\text{ cm.}$$

$$d = + \frac{\lambda}{2\pi}\frac{\pi}{5} \bmod \lambda$$

$2\pi$

$= \frac{1}{2}$ mod 5cm

0.5, 5.5, 10.5, 15.5,

$\lambda = 7$ cm

$\phi' =$

3.5, 10.5, 17.5,

24.5 –

30cm

7

5

4 6